9 Stella Octangula

Themes	Stellation and duality of regular polyhedra.
Vocabulary	Stellation, union, intersection, duality.
Synopsis	Either stellate an octahedron or the middle triangles of the faces of the double edge length tetrahedron. Observe the fact that it is the union of two double edge length tetrahedra. Use tape to reveal the cube dual of the octahedron.

Overall structure	**Previous**	**Extension**
1 Use, Safety and the Rhombus		
2 Strips and Tunnels		
3 Pyramids	**X**	
4 Regular Polyhedra	**X**	
5 Symmetry		
6 Colour Patterns		
7 Space Fillers		
8 Double edge length tetrahedron (can provide the double edge length tetrahedron which can be extended to the stella octangula in this activity)	**X**	
9 Stella Octangula		
10 Stellated Polyhedra and Duality (generalise to other polyhedra)		**X**
11 Faces and Edges		
12 Angle Deficit		
13 Torus		

Layout

The activity description is in this font, with possible speech or actions as follows:

Suggested instructor speech is shown here with

possible student responses shown here.
'Alternative responses are shown in quotation marks'.

1a Starting with the double edge length tetrahedron

Figure 1 The double edge length tetrahedron

If you are continuing from the double edge length tetrahedron activity, as in figure 1, you already have a double edge tetrahedron preferably in a single colour.

Demonstrate how to add three triangles coming together at a vertex to stellate the mid triangle of each edge. This should be 12 triangles ideally of one single colour. The outcome is shown in figure 2, and can be seen as an extension and application of the way an octahedron can be coloured with two colours in the Colour Patterns activity (6).

Figure 2 The stella octangula

The method of tying the extra three triangles is

- To make four sets of three triangles, tied together to give a tetrahedron or stellating pyramid with one face missing. Use all one colour if possible for each of the four pyramids. This can be done by the class.

  We need four groups to each make a three sided pyramid without a base by tying three triangles together, and everyone use this colour triangle for all the pyramids.

- Then untie the 3 middle knots around the four centre faces of the double edge length tetrahedron. Demonstrate on one the face and have students complete the task.

  We will attach this over the middle triangles of the faces, so first we untie all the middle knots around the centre triangles of each face, but we do not untie the end ones.

- Attach a stellating pyramid over each the centre triangle of each face as in figure 3. A knot is tied connecting two middle laces from the double edge length tetrahedron and one middle from the added three triangles.

Figure 3 Tying three laces together

Once the mid triangles of the face have been stellated:

Attach the pyramids by tying on the middle laces only. You have to tie 3 laces together, so the two in one hand, squeezing them together and one in the other, and tie normally like this.

1b Starting with an octahedron (instead of step 1a)

This procedure will make a stella octangula like the one in figure 2, and also a separate octahedron.

Put out a pile of 12 triangles all of one colour and another pile with 12 triangles of one other colour.

We need 4 groups to send one person up to the front. Each person take three triangles of one colour and three triangles of the other colour for your group.

Each group make two pyramids, one of one colour and one of the other colour.

Now we just need one group to make an octahedron using other colours.

Once all shapes have been built. Place the octahedron centrally on a face and say

We are going to attach the pyramids together around the octahedron. So each face of the octahedron will have a pyramid on it.

We are also going to make the colours of pyramids alternate. So each pyramid of one colour will only share an edge with pyramids of the other colour.

We will start with the four pyramids this colour.

Place one the four pyramids on the top face of the octahedron, and the other three on the floor meeting up with the 3 overhanging faces of the octahedron.

Now take a pyramid of the other colour and hold it over one of the uncovered faces of the octahedron.

We will just tie the adjacent pyramids together first by the laces in the middle of their edges.

Once this is done, leaving the octahedron on the floor say:

Tie the other two pyramids on the uncovered faces the same way. We are only tying pyramids to each other not to the octahedron.

Once they are tied lift the assembly up and remove the octahedron. Replace the assembly flat on the ground in the same position and have the students tie the other laces and then add and finally tie the last pyramid.

2 Naming and investigating the stella octangula.

See figure 2. Hold the stella octangula up on a vertex, and ask students what they can see:

What shapes can you see?

'A pyramid pointing up',
'a big tetrahedron'

How many large tetrahedra are there?

One pointing up one pointing down

What colours are they?

Blue and clear

What are their edge lengths

both 2

Point them both out

This ensures students are not confusing them with the single edge length tetrahedra that stellate the shape. Now check the vocabulary word 'intersection':

What is the word for where two things overlap, like these edge length 2 tetrahedra?

Intersection

What shape is the intersection of these two edge length 2 tetrahedra?

Octahedron

What is the difference between the overall shape, and the intersection shape, the octahedron?

Little pyramids

Yes edge length 1 pyramids.

How many?

8

Every face of the octahedron has a pyramid on it. We call this stellation

This shape is a stellated octahedron and we also give it the name stella octangula.

If your curriculum covers union and intersection then you can discuss union in this context as well. The overall shape is the union of two large tetrahedra.

3 The cube

Figure 4 The cube

Placing the stella octangula on four of its attached pyramids. A cube can be outlined with masking tape as shown in figure 4. Make the first edge with masking tape and have the students continue. It is useful to have three or four rolls of tape for this. Ensure the tape is good quality otherwise it will tear too much.

4 Discussing duality

The dual relationship between the octahedron and cube is that the vertices of one shape lie over the midpoints of the faces of the other.

Hold up the octahedron alongside the cube and ask

> Look at the cube. Can anyone see which way the original octahedron fits in the middle? Can anyone show us?

If no one comes forward, ask:

> Which way do you think it would fit? With a horizontal face or standing up on a vertex?

Standing up

This provides an opportunity to identify locations of the faces, edges and vertices of the octahedron in the stella octangula.

> Can anyone show us where the faces, edges or vertices of the octahedron are?

The faces of the octahedron are covered by the bases of the 8 pyramids. Four facing up and four facing down. Holding a loose octahedron directly above, or beside the stella octangula at the correct angle can be used to hint at and explain this.

> If this octahedron slides into the middle of the stella octangula, where will its edges be on the stella octangula?
>
> Who can run their hands along them to show us?

Demonstrate if necessary.

The vertices of the octahedron are in the midpoints of the faces of the cube. This can be hinted at and demonstrated as above. Summarize.

> Where are the vertices of the octahedron?

In the middle of the faces of the cube

Now we connect the discussion to the numbers of faces and vertices. You may want to note these answers on the board to refer back to.

> How many faces does the cube have?

6

Yes, four on the sides, one on top and one underneath

Hold up the octahedron on a vertex, and ask:

How many vertices does the octahedron have?

6

Touch the vertices as you mention them.

Yes, four around the middle, one on top and one underneath.

How many vertices does the cube have?

8

Yes, four on top and four below them.

Hold up the octahedron

How many faces does the octahedron have?

8

Yes four facing up and four facing down

Now we summarize the dual relationship

The 6 vertices of the octahedron are at the midpoints of the 6 faces of the cube.

Where are the faces of the octahedron

Under each pyramid

The 8 faces of the octahedron are under each of the 8 vertices of the cube.

Vertices of one shape correspond to faces of the other and faces of one shape correspond to vertices of the other.

We say that the octahedron and cube are duals of each other.

When we stellate a shape the apexes of each pyramid added becomes a vertex over the midpoint of an original face.

Therefore when we stellate a suitable shape with room for the pyramids, we can find its dual by connecting the apexes with new edges (that we made with tape). The apexes are the vertices of the dual, and the new edges are the edges of the dual and they outline the faces of the dual.

